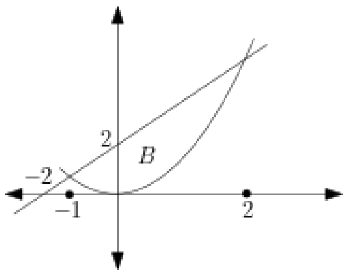
MTS225 İNTEGRAL HESAP ARA SINAV 2020 ÇÖZÜMLER

1. i)



$x^2 = x + 2 \quad x = -1, 2$

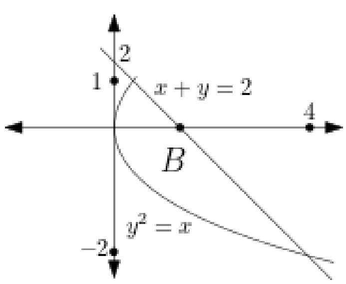
$B : -1 \leq x \leq 2 \quad x^2 \leq y \leq x + 2$

$$\text{Kütle} = \iint_B y\,dA = \int_{-1}^{2}\int_{x^2}^{x+2} y\,dy\,dx = \int_{-1}^{2} \tfrac{1}{2}((x+2)^2 - x^4)\,dx = \frac{36}{5}$$

$$M_y = \iint_B xy\,dA = \int_{-1}^{2}\int_{x^2}^{x+2} xy\,dy\,dx = \int_{-1}^{2} \tfrac{1}{2}(x(x+2)^2 - x^5)\,dx = \frac{45}{8}$$

$$\bar{x} = \frac{M_y}{\text{Kütle}} = \frac{\frac{45}{8}}{\frac{36}{5}} = \tfrac{25}{32}$$

ii)



$y^2 = 2 - y \quad y = -2, 1$

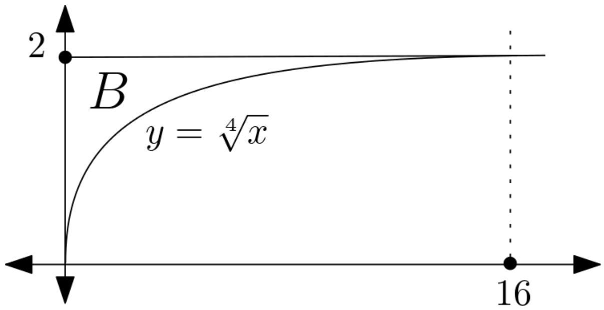
$B : -2 \leq y \leq 1 \quad y^2 \leq x \leq 2 - y$

$$\text{Kütle} = \iint_B x\,dA = \int_{-1}^{2}\int_{y^2}^{2-y} x\,dx\,dy = \int_{-2}^{1} \tfrac{1}{2}((2-y)^2 - y^4)\,dy = \frac{36}{5}$$

$$M_x = \iint_B xy\,dA = \int_{-1}^{2}\int_{y^2}^{2-y} xy\,dx\,dy = \int_{-2}^{1} \tfrac{1}{2}(y(2-y)^2 - y^5)\,dy = -\frac{45}{8}$$

$$\bar{y} = \frac{M_x}{\text{Kütle}} = \frac{-\frac{45}{8}}{\frac{36}{5}} = -\tfrac{25}{32}$$

2. i)



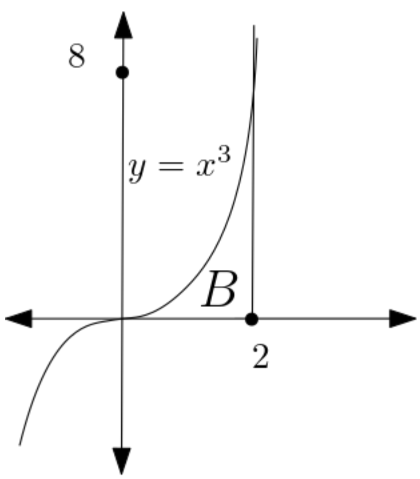
$$\int_0^{16}\left(\int_{\sqrt[4]{x}}^{2}\frac{dy}{1+y^5}\right)dx = \iint_B \frac{1}{1+y^5}\,dA$$

$B: 0 \le x \le 16,\ \sqrt[4]{x} \le y \le 2$

$B: 0 \le y \le 2,\ 0 \le x \le y^4$

$$\iint_B \frac{1}{1+y^5}\,dA = \int_0^2\left(\int_0^{y^4}\frac{1}{1+y^5}dx\right)dy = \int_0^2\left(\frac{x}{1+y^5}\bigg|_0^{y^4}\right)dy$$
$$= \int_0^2 \frac{y^4}{1+y^5}\,dy = \frac{\ln(1+y^5)}{5}\bigg|_0^2 = \frac{\ln 33}{5}$$

ii)



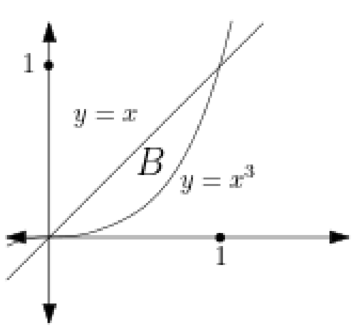
$$\int_0^8\left(\int_{\sqrt[3]{y}}^{2} e^{x^4}\,dx\right)dy = \iint_B e^{x^4}\,dA$$

$B: 0 \le y \le 8,\ \sqrt[3]{y} \le x \le 2$

$B: 0 \le x \le 2,\ 0 \le y \le x^3$

$$\iint_B e^{x^4}\,dA = \int_0^2\left(\int_0^{x^3} e^{x^4}dy\right)dx = \int_0^2\left(ye^{x^4}\Big|_0^{x^3}\right)dx = \int_0^2 x^3e^{x^4}\,dx = \frac{1}{4}e^{x^4}\bigg|_0^2 = \frac{e^{16}-1}{4}$$

iii)

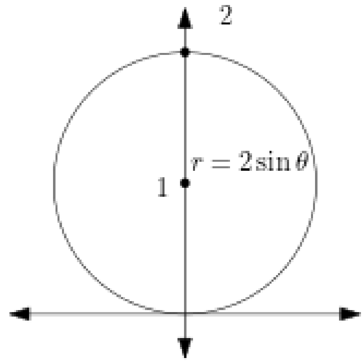


$$\int_0^1\left(\int_y^{\sqrt[3]{y}}\frac{dx}{1+x^4}\right)dy = \iint_B \frac{1}{1+x^4}\,dA$$

$B: 0 \le y \le 1,\ \sqrt[3]{y} \le x \le y$

$B: 0 \le x \le 1,\ x^3 \le y \le x$

$$\iint_B \frac{1}{1+x^4}\,dA = \int_0^1\left(\int_{x^3}^{x}\frac{1}{1+x^4}dy\right)dx = \int_0^1\left(\frac{y}{1+x^4}\bigg|_{x^3}^{x}\right)dx$$
$$= \int_0^1\frac{x}{1+x^4}\,dx - \int_0^1\frac{x^3}{1+x^4}\,dx = \left(\frac{1}{2}\operatorname{Arctan}(x^2) - \frac{1}{4}\ln(1+x^4)\right)\bigg|_0^1 = \frac{\pi}{8} - \frac{\ln 2}{4}$$



3. i) $z = 2x^2 + 2y^2 = 4y \quad x^2 + y^2 = 2y \quad r = 2\sin\theta$
$B: 0 \le \theta \le \pi, \quad 0 \le r \le 2\sin\theta$

$$\text{Hacim} = \iint_B (4y - 2x^2 - 2y^2)\, dA = \int_0^{\pi}\int_0^{2\sin\theta} (4r\sin\theta - 2r^2)\, r\, dr\, d\theta$$
$$= \int_0^{\pi} \left(\frac{5}{6}\sin^4\theta\right) d\theta = \frac{5}{6}\left(\frac{3\theta}{8} - \frac{1}{4}\sin(2\theta) + \frac{1}{32}\sin(4\theta)\right)\Bigg|_0^{\pi} = \frac{5\pi}{16}$$

ii) $z = 4 - x^2 - y^2,\ z = 0$ arasında, $B:\ x^2 + y^2 = 2y$ içinde
$B: 0 \le \theta \le \pi, \quad 0 \le r \le 2\sin\theta$

$$\text{Hacim} = \iint_B (4 - x^2 - y^2 - 0)\, dA = \int_0^{\pi}\int_0^{2\sin\theta} (4 - r^2)\, r\, dr\, d\theta$$
$$= \int_0^{\pi} \left(8\sin^2\theta - 4\sin^4\theta\right) d\theta = \left(\frac{5\theta}{2} - \sin(2\theta) - \frac{1}{8}\sin(4\theta)\right)\Bigg|_0^{\pi} = \frac{5\pi}{2}$$

iii) $x^2 + y^2 - 4y + 9 = 9 - x^2 - y^2$ arasında, $B:\ x^2 + y^2 = 2y$ içinde
$B: 0 \le \theta \le \pi, \quad 0 \le r \le 2\sin\theta$

$$\text{Hacim} = \iint_B (4y - 2x^2 - 2y^2)\, dA = \int_0^{\pi}\int_0^{2\sin\theta} (4r\sin\theta - 2r^2)\, r\, dr\, d\theta$$
$$= \int_0^{\pi} \frac{8}{3}\sin^4\theta\, d\theta = \left(\theta - \frac{2}{3}\sin(2\theta) + \frac{1}{12}\sin(4\theta)\right)\Bigg|_0^{\pi} = \pi$$