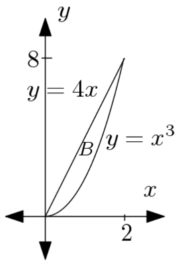
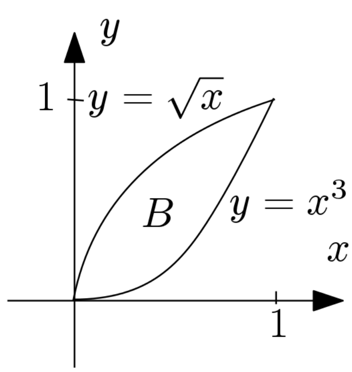
MTS 225 İntegral Hesap Ara Sınav (2019) Çözümler

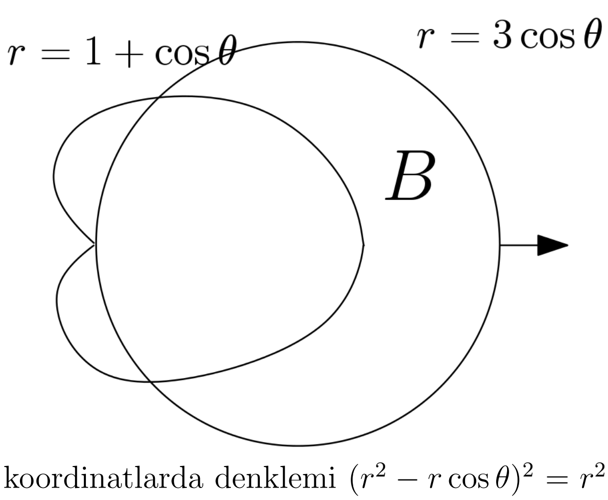


1. $B:\ 0 \le y \le 8,\ \sqrt[3]{y} \le x \le \frac{y}{4}$ (II. Tip) ve aynı zamanda $B:\ 0 \le x \le 2,\ x^3 \le y \le 4x$ (I. Tip)
Fubini nin Teoreminden,

$$\int_0^8 \left(\int_{\sqrt[3]{y}}^{\frac{y}{4}} \frac{1}{1+x^4}\, dx \right) dy = \int_B \frac{1}{1+x^4}\, dA = \int_0^2 \left(\int_{x^3}^{4x} \frac{1}{1+x^4}\, dy \right) dx$$
$$= \int_0^2 \left(\frac{4x}{1+x^4} - \frac{x^3}{1+x^4} \right) dx = \left(2\operatorname{Arctan}(x^2) - \tfrac{1}{4}\ln(1+x^4) \right)\Big|_0^2 = 2\operatorname{Arctan} 4 - \tfrac{1}{4}\ln 17$$



2. $B: 0 \le x \le 1, \quad x^3 \le y \le \sqrt{x}$ (I. Tip olarak)
Kütle$=\int_B \rho(x,y)\, dA = \int_B x\, dA \overset{Fubini}{=} \int_0^1 \int_{x^3}^{\sqrt{x}} x\, dy\, dx = \int_0^1 \left(x^{\frac{3}{2}} - x^4 \right) dx = \dfrac{1}{5}$
$\int_B y\rho(x,y)\, dA = \int_B yx\, dA \overset{Fubini}{=} \int_0^1 \left(\int_{x^3}^{\sqrt{x}} xy\, dy \right) dx = \int_0^1 \dfrac{1}{2}\left(x^2 - x^7 \right) dx = \dfrac{5}{48},$
$\bar{y} = \dfrac{\int_B y\rho(x,y)\, dA}{\int_B \rho(x,y)\, dA} = \frac{25}{48}$

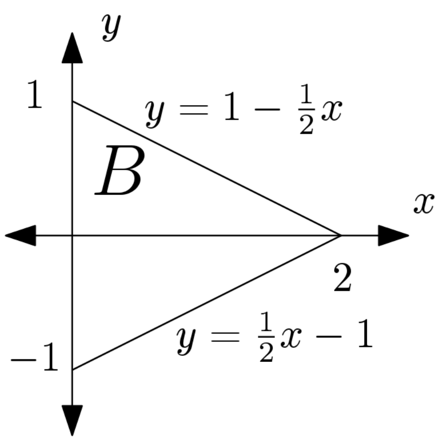


3. $B:\ (x^2+y^2-x)^2 = x^2+y^2$ kardiyoidin kutupsal koordinatlarda denklemi $(r^2 - r\cos\theta)^2 = r^2$ den $r = 1+\cos\theta$ (aslında bir de $r = -1+\cos\theta$ eğrisi

bulunuyor ama ikisi aynı eğri!), çemberin denklemi: $r = 3\cos\theta$ olur.
Bu eğriler $\theta = \pm\frac{\pi}{3}$ noktalarında kesişir.
($-\frac{\pi}{3} \le \theta \le \frac{\pi}{3}$ için $1+\cos\theta \le 3\cos\theta$ olduğundan) $B:\ -\frac{\pi}{3} \le \theta \le \frac{\pi}{3}, \quad 1+\cos\theta \le r \le 3\cos\theta$ olur.
(diğer tüm koşullar da sağlanıyor) İki katlı integrallerde değişken değişikliği formülünden

$$\int_B |y|\,dA = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(\int_{1+\cos\theta}^{3\cos\theta} |r\sin\theta|\, r\,dr \right) d\theta = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{3}\left((3\cos\theta)^3 - (1+\cos\theta)^3)\right) |\sin\theta|\,d\theta$$

$$= \frac{2}{3}\int_0^{\frac{\pi}{3}} \left(((3\cos\theta)^3 - (1+\cos\theta)^3)\sin\theta\right) d\theta \overset{u=\cos\theta}{=} \frac{2}{3}\int_{\frac{1}{2}}^{1} (27u^3 - (1+u)^3)\,du = \frac{115}{48}$$



4. Hacim$=\int_B (4 - x^2 - y^2 - 0)\,dA$
$B: 0 \le x \le 2, \quad \frac{1}{2}x - 1 \le y \le 1 - \frac{1}{2}x$
(diğer tüm koşullar da sağlanıyor) Fubini nin Teoreminden:

$$\int_B (4 - x^2 - y^2 - 0)\,dA = \int_0^2 \left(\int_{\frac{1}{2}x-1}^{1-\frac{1}{2}x} (4 - x^2 - y^2)\,dy \right) dx = \int_0^2 \left(4y - x^2y - \tfrac{1}{3}y^3\right)\Big|_{\frac{1}{2}x-1}^{1-\frac{1}{2}x} dx$$

$$= \int_0^2 \left((2-x)(4-x^2) - \tfrac{1}{3}((1-\tfrac{1}{2}x)^3 - (\tfrac{1}{2}x-1)^3)\right) dx$$

$$\overset{u=2-x}{=} \int_0^2 (u^2(4-u) - \tfrac{1}{12}u^3)\,du = \tfrac{19}{3}$$