Sorular

1) $f(x) : \frac{x^2+x}{x-1} \qquad x>0$

$\qquad\quad \frac{\sin 3x}{x} \qquad x<0$

$\lim\limits_{x\to 0} f(x) = ?$

2) $\lim\limits_{x\to 0^+} \frac{\sin x}{\sqrt{x}} = ?$ $\lim\limits_{x\to 0^+} \frac{\sin\sqrt{x}}{x} = ?$

3) $\lim\limits_{x\to \mp\infty} \sqrt{x^2+3x}+x = ?$

4) $f(x)$ çift fonksiyon olsun.

$$\lim_{x\to 0} f(x) = L \Longleftrightarrow \lim_{x\to 0^+} f(x) = L$$ olduğunu gösteriniz.

5) $\lim\limits_{x\to +\infty} \sqrt[3]{x^3+x^2} - x = ?$

6) $\lim\limits_{x\to 0} \frac{\sin(\sin x)}{x} = ?$

7) $f(x)$ tek fonksiyon olsun.

$$\lim_{x\to a} f(x) = L \Longleftrightarrow \lim_{x\to -a} f(x) = -L \qquad \text{ve} \lim_{x\to a^+} f(x) = L \Longleftrightarrow$$ $\lim\limits_{x\to -a^-} f(x) = -L$ olduğunu gösteriniz.

8) $\lim\limits_{x\to 5} \frac{x^2-3x+1}{x^2-6x+5} = ?$, $\lim\limits_{x\to +\infty} \frac{x^2-3x+1}{x^2-6x+5} = ?$

9), $\lim\limits_{x\to \pm\infty} \frac{\sqrt{x^2+x}+1}{2x-1} = ?$

Çözümler

1) $f(x) : \frac{x^2+x}{x-1} \qquad x>0$

$\qquad\quad \frac{\sin 3x}{x} \qquad x<0$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{x^2+x}{x-1} = 0$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{\sin 3x}{x} = \lim\limits_{x\to 0^-} 3.\frac{\sin 3x}{3x} = \lim\limits_{x\to 0^-} 3. \lim\limits_{3x\to 0^-} \frac{\sin 3x}{3x} = \lim\limits_{x\to 0^-} 3. \lim\limits_{h\to 0^+} \frac{\sin -h}{-h} = 3$ $(3x=-h)$

2)a) $\lim\limits_{x\to 0^+} \frac{\sin x}{\sqrt{x}} = \lim\limits_{x\to 0^+} \sqrt{x}\frac{\sin x}{\sqrt{x}\sqrt{x}} = \lim\limits_{x\to 0^+} \sqrt{x}\frac{\sin x}{x} = \lim\limits_{x\to 0^+} \sqrt{x} \lim\limits_{x\to 0^+} \frac{\sin x}{x} = 0.1 = 1$

b) $\lim\limits_{x\to 0^+} \frac{\sin\sqrt{x}}{x} = \lim\limits_{x\to 0^+} \frac{\sin\sqrt{x}}{\sqrt{x}\sqrt{x}} = \lim\limits_{x\to 0^+} \frac{\sin\sqrt{x}}{\sqrt{x}} \lim\limits_{x\to 0^+} \frac{1}{\sqrt{x}} = 1.+\infty = +\infty$

3)a) $\lim\limits_{x\to -\infty} \sqrt{x^2+3x}+x = \lim\limits_{x\to -\infty} (\sqrt{x^2+3x}+x)(\frac{\sqrt{x^2+3x}-x}{\sqrt{x^2+3x}-x}) = \lim\limits_{x\to -\infty} (\frac{x^2+3x-x^2}{\sqrt{x^2+3x}-x}) = \lim\limits_{x\to -\infty} \frac{3x}{\sqrt{x^2+3x}-x} = \lim\limits_{x\to -\infty} \frac{3x}{|x|(\sqrt{1+\frac{3}{x^2}})-x} = \lim\limits_{x\to -\infty} \frac{3x}{-x(\sqrt{1+\frac{3}{x^2}})-x} = \lim\limits_{x\to -\infty} \frac{3x}{-x(\sqrt{1+\frac{3}{x^2}}-1)} = -\frac{3}{2}$

b) $\lim\limits_{x\to +\infty} \sqrt{x^2+3x}+x = \lim\limits_{x\to +\infty} (\sqrt{x^2+3x}+x)(\frac{\sqrt{x^2+3x}-x}{\sqrt{x^2+3x}-x}) = \lim\limits_{x\to +\infty} (\frac{x^2+3x-x^2}{\sqrt{x^2+3x}-x}) = \lim\limits_{x\to +\infty} \frac{3x}{\sqrt{x^2+3x}-x} = \lim\limits_{x\to +\infty} \frac{3x}{|x|(\sqrt{1+\frac{3}{x^2}})-x} = \lim\limits_{x\to +\infty} \frac{3x}{x(\sqrt{1+\frac{3}{x^2}})-x} = \lim\limits_{x\to +\infty} \frac{3x}{x(\sqrt{1+\frac{3}{x^2}}-1)} = \frac{3}{0} = +\infty$

4) $f(x)$ çift fonksiyon olsun.

$$\lim_{x\to 0} f(x) = L \Longleftrightarrow \lim_{x\to 0^+} f(x) = L$$

$\Longrightarrow :$ $\lim\limits_{x\to 0} f(x) = L \Rightarrow$Sağdan ve soldan limit vardır ve eşittir.$\Rightarrow \lim\limits_{x\to 0^+} f(x) = L$ dir.

$\Longleftarrow: t=-x \Rightarrow -t=x \Rightarrow x\to 0^- \Rightarrow t\to 0^+$

$f(-x)=f(x)$ ise $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{t\to 0^+} f(-t) = \lim\limits_{t\to 0^+} f(t) = \lim\limits_{x\to 0^+} f(x) = L$ sağdan soldan limit var ve eşit olduğundan $\lim\limits_{x\to 0} f(x)=L$ dir.

5) $\lim\limits_{x\to+\infty} \sqrt[3]{x^3+x^2}-x = \lim\limits_{x\to+\infty} \sqrt[3]{x^3+x^2} - x\left(\frac{\sqrt[3]{(x^3+x^2)^2}+\sqrt[3]{x^3+x^2}.x+x^2}{\sqrt[3]{(x^3+x^2)^2}+\sqrt[3]{x^3+x^2}.x+x^2}\right) =$
$\lim\limits_{x\to+\infty} \frac{x^3+x^2-x^3}{\sqrt[3]{(x^3+x^2)^2}+\sqrt[3]{x^3+x^2}.x+x^2} = \lim\limits_{x\to+\infty} \frac{x^2}{\sqrt[3]{(x^3+x^2)^2}+\sqrt[3]{x^3+x^2}.x+x^2} = \lim\limits_{x\to+\infty} \frac{x^2}{|x|\sqrt[3]{(1+\frac{2}{x}+\frac{1}{x^2}}+|x|\sqrt[3]{1+\frac{1}{x}}.x+x^2} =$

$= \lim\limits_{x\to+\infty} \frac{x^2}{x\sqrt[3]{(1+\frac{2}{x}+\frac{1}{x^2}}+x\sqrt[3]{1+\frac{1}{x}}.x+x^2} = \lim\limits_{x\to+\infty} \frac{x^2}{x^2(\sqrt[3]{(1+\frac{2}{x}+\frac{1}{x^2}}+\sqrt[3]{1+\frac{1}{x}}.+1)} = \frac{1}{3}$

6) $\lim\limits_{x\to 0} \frac{\sin(\sin x)}{x} = \lim\limits_{x\to 0} \frac{\sin(\sin x)}{x}\frac{\sin x}{\sin x} = \lim\limits_{x\to 0}\frac{\sin(\sin x)}{\sin x}\lim\limits_{x\to 0}\frac{\sin x}{x} = \lim\limits_{\sin x\to 0}\frac{\sin(\sin x)}{\sin x}\lim\limits_{x\to 0}\frac{\sin x}{x}$ $(\sin x = t, x\to 0, \sin x\to 0$ ve $x\neq 0,\ x\in(-\frac{\pi}{2},\frac{\pi}{2})$ için $\sin x\neq 0)$

$=\lim\limits_{t\to 0}\frac{\sin(t)}{\sin t}\lim\limits_{x\to 0}\frac{\sin x}{x} = 1$

7) $f(x)$ tek fonksiyon olsun.

$\lim\limits_{x\to a} f(x) = L \Longleftrightarrow \lim\limits_{x\to -a} f(x) = -L$ ve $\lim\limits_{x\to a^+} f(x) = L \Longleftrightarrow \lim\limits_{x\to -a^-} f(x) = -L$ olduğunu gösteriniz.

$f(x)$ tek fonksiyon ise $f(-x)=-f(x)$ ($t=-x$ değişken değişikliğini kullanacağız..)

a) $\lim\limits_{x\to a} f(x) = L$ olsun. $\lim\limits_{x\to -a} f(x) = \lim\limits_{t\to a} f(-t) = -\lim\limits_{t\to a} f(t) = -\lim\limits_{x\to a} f(x) = -L$

b) $\lim\limits_{x\to a^+} f(x) = L$ olsun. $\lim\limits_{x\to -a^-} f(x) = \lim\limits_{\substack{-x\to a^+\\ t=-x}} f(x) = \lim\limits_{t\to a^+} f(-t) = -\lim\limits_{t\to a^+} f(t) = -\lim\limits_{x\to a^+} f(x) = -L$

8) $a) \lim\limits_{x\to 5}\frac{x^2-3x+1}{x^2-6x+5} =?, b) \lim\limits_{x\to+\infty}\frac{x^2-3x+1}{x^2-6x+5} =?$

a) 1.yol:limit teoremi ile $\lim\limits_{x\to 5^+}\frac{x^2-3x+1}{x^2-6x+5} = \lim\limits_{x\to 5^+}\frac{x^2-3x+1}{x-1}\frac{1}{x-5} = \frac{11}{4}.(+\infty) = +\infty$(limitteoremi ile

$$\lim\limits_{x\to 5^-}\frac{x^2-3x+1}{x^2-6x+5} = \lim\limits_{x\to 5^-}\frac{x^2-3x+1}{x-1}\frac{1}{x-5} = \frac{11}{4}.(-\infty) = -\infty$$

2.yol:sonsuz limit tanımından : $x^2-3x+1=0 \Rightarrow 3 > \frac{3\pm\sqrt{5}}{2} > 0$ ve $f(x)=\frac{x^2-3x+1}{x^2-6x+5}$ olmak üzere

$x>0$ için $f(x)>0$ ve $3<x<5$için $f(x)<0 \Rightarrow \lim\limits_{x\to 5}\frac{1}{f(x)} = 0$ $\quad \lim\limits_{x\to 5^+}\frac{x^2-3x+1}{x^2-6x+5} = +\infty$ ve $\lim\limits_{x\to 5^-}\frac{x^2-3x+1}{x^2-6x+5} = -\infty$

b) $\lim\limits_{x\to+\infty}\frac{x^2-3x+1}{x^2-6x+5} = 1$

9) $\lim\limits_{x\to\pm\infty}\frac{\sqrt{x^2+x}+1}{2x-1} =?$

a) $\lim\limits_{x\to+\infty}\frac{\sqrt{x^2+x}+1}{2x-1} = \frac{1}{2},\ \lim\limits_{x\to-\infty}\frac{\sqrt{x^2+x}+1}{2x-1} = -\frac{1}{2}$